Hard exclusive production of two pions: Dipion mass distributions in $\gamma^* N \to \pi\pi N$ and $\gamma^*\gamma \to \pi\pi$

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The leading twist parametrization of dipion mass distribution in hard exclusive reactions is proposed. Its parameters are related to quark distributions (usual and skewed) in the pion and to distributions amplitudes of mesons (π, ρ, etc.). We show that measurements of the shape of dipion mass distribution in hard exclusie reactions can give important information about partonic structure of the pion. The expression for the amplitude of the reaction $\gamma^*\gamma \to \pi\pi$ near the threshold in terms of singlet quark distribution in the pion is presented.

1. Dipion mass distribution in $\gamma^* + p \to \pi^+\pi^- + p$

Recently it became possible to measure with good precision two pion hard exclusive production in the deeply virtual photon fragmentation region in the range of dipion masses $0.4 \leq m_{\pi\pi} \leq 1.5$ GeV (see talks at this conference [1] and refs. [2, 3]).

Owing to QCD factorization theorem for hard exclusive reaction [4] the dependence of the amplitude of the reactions

$$\gamma_L^* + T \to \pi\pi + T' \tag{1}$$

on the dipion mass $m_{\pi\pi}$ factorizes at leading order into the universal (independent of the target) factor:

$$\int_0^1 \frac{dz}{z}\, \Phi^I(z,\zeta,m_{\pi\pi};Q^2)\ . \tag{2}$$

Here $\Phi^I(z,\zeta,m_{\pi\pi};Q^2)$ is two-pion light cone distribution amplitude (2πDA), which depends on z–longitudinal momentum carried by the quark, ζ characterizing the distribution of longitudinal momentum between the two pions, and the invariant mass of produced pions $m_{\pi\pi}$, superscript I stands for isospin of produced pions ($I = 0, 1$). The dependence on the virtuality of the incident photon Q^2 is governed by the evolution equation.

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The 2πDA's were introduced recently in [5] in the context of the QCD description of the process $\gamma^*\gamma \to 2\pi$, its detailed properties were studied in [6], we refer to these papers for details.

For the process (1) at small x_{Bj} (see *e.g.* recent measurements [2, 3]) the production of two pions in the isoscalar channel is strongly suppressed relative to the isovector channel, because the former is mediated by C-parity odd exchange. At asymptotically large Q^2 QCD predicts the following simple form for the isovector 2πDA [7, 6]:

$$\Phi_{as}^{I=1}(z,\zeta,m_{\pi\pi}) = 6z(1-z)(2\zeta-1)F_\pi(m_{\pi\pi})\ , \tag{3}$$

where $F_\pi(m_{\pi\pi})$ is the pion e.m. form factor measured with high precision in low energy experiments [8]. From eqs. (2,3) we conclude that at asymptotically large Q^2 QCD predicts unambiguously the shape of the dipion mass distribution:

$$\frac{dN(m_{\pi\pi})}{dm_{\pi\pi}^2} \propto \left(1-\frac{4m_\pi^2}{m_{\pi\pi}^2}\right)^{3/2} |F_\pi(m_{\pi\pi})|^2\,. \tag{4}$$

At non-asymptotic Q^2 the 2πDA deviates from its asymptotic form (3). This deviation can be described by a few parameters which can be related to quark distributions (skewed and usual) in the pion and to distribution amplitudes of mesons ($\pi\rho$, etc.), for details see [6]. In this case we propose the following parametrization of dipion mass

distribution:

$$
\begin{aligned}
\frac{dN(m_{\pi\pi})}{dm^2_{\pi\pi}} &\propto \left(1-\frac{4m_\pi^2}{m^2_{\pi\pi}}\right)^{3/2} |F_\pi(m_{\pi\pi})|^2 \\
&\times \left(1+D_1(m_{\pi\pi},Q^2)\right)^2 \qquad (5)\\
&+ \frac{3}{7}\left(1-\frac{4m_\pi^2}{m^2_{\pi\pi}}\right)^{7/2} |D_2(m_{\pi\pi},Q^2)|^2,
\end{aligned}
$$

where the functions $D_{1,2}(m_{\pi\pi},Q^2)$ describe the deviation of the dipion mass distribution from its asymptotic form (4) and can be parametrized in the form:

$$
\begin{aligned}
D_1(m_{\pi\pi},Q^2) &= C_1(Q^2)e^{b_1 m^2_{\pi\pi}} \\
&- \frac{6m_\pi^2}{m^2_{\pi\pi}}\, C_2(Q^2)e^{b_2 m^2_{\pi\pi}} \qquad (6)\\
D_2(m_{\pi\pi},Q^2) &= C_2(Q^2)e^{b_3 m^2_{\pi\pi}}\,.
\end{aligned}
$$

The dependence of $C_{1,2}(Q^2)$ on Q^2 is governed by the QCD evolution and in leading order is given by:

$$
C_{1,2}(Q^2) = C_{1,2}(\mu_0)\left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu_0)}\right)^{50/(99-6n_f)}\,. \qquad (7)
$$

With increasing of Q^2 the parameters $C_{1,2}(Q^2)$ go logarithmically to zero and one reproduces the asymptotic formula (4).

The parameters b_i are Q^2 independent but, in principle, $m_{\pi\pi}$ dependent. The latter dependence is fixed by $\pi\pi$ scattering phase shifts, see [6].

In derivation of eq. (6) we neglect the production of pions in G-waves and higher because it is suppressed by powers of $1/\log(Q^2)$.

Interestingly that the parameters $C_{1,2}(\mu_0)$ and b_i can be related to important parameters of quark distributions in the pion and meson distribution amplitudes [6]. Using soft pion theorems for 2πDA's [6] one can express the second Gegenbauer moment of pion distribution amplitude in terms of $C_{1,2}(\mu_0)$:

$$
a_2^{(\pi)}(\mu_0) = C_1(\mu_0) + C_2(\mu_0)\,. \qquad (8)
$$

If one additionally uses the dispersion relation derived in [6] one gets the expression for the second Gegenbauer moment of ρ-meson distribution amplitude:

$$
a_2^{(\rho)}(\mu_0) \approx C_1(\mu_0)\exp(b_1 m_\rho^2)\,. \qquad (9)
$$

The crossing relations [6] (see also [9, 10]) allow to relate the parameter $C_2(\mu_0)$ to the third Mellin moment of quark distribution in the pion at normalization point μ_0

$$
\int_0^1 dx\; x^2(u^{\pi^+}(x) - \bar u^{\pi^+}(x)) = \frac{6}{7}\, C_2(\mu_0)\,. \qquad (10)
$$

In analysis of experiments on two pion diffractive production off nucleon (see e.g. [2, 3]) the dipion mass distribution is usually fitted by, for example, Söding parametrization, which takes into account rescattering of produced pions on final nucleon. Let us note however that in the case of hard ($Q^2\to\infty$) diffractive production the final state interaction of pions with residual nucleon is suppressed by powers of $1/Q^2$ relative to the leading twist amplitude. Also the parameters of Söding (or Ross-Stodolsky) parametrization are not related to any fundamental parameter of QCD. Here we proposed alternative leading-twist parametrization (6) describing the so-called "skewing" of two pion spectrum. The advantage of our parametrization is that its parameters can be related to fundamental quantities in QCD: quark distributions in the pion and meson distribution amplitudes.

2. Reaction $\gamma^*\gamma\to\pi\pi$ near threshold

Another example of hard exclusive reaction where the 2πDA enters is the reaction $\gamma^*\gamma\to\pi\pi$ with Q^2 of the photon large and the invariant mass of produced pions is small. In refs. [5, 11] it was demostrated that this process is amenable to QCD description. The amplitude in the leading order can be expressed in terms of isoscalar 2πDA [5]:

$$
M_{\mu\nu} = \sum_f e_f^2\; g_{\mu\nu}^{\perp}\int_0^1 \frac{dz}{z}\,\Phi^{I=0}(z,\zeta,m_{\pi\pi};Q^2)\,. \qquad (11)
$$

Using the crossing relations and soft pion theorems we derive the expression for the amplitude $M_{\mu\nu}$ close to the threshold $m_{\pi\pi}=2m_\pi$ in terms

of singlet quark distribution in the pion. To this end we make one additional approximation: that production of two pions in the state with angular momentum l is dominated by the operators with conformal spin $l-1$ and $l+1$. The higher Q^2 the more this approximation is justified, see for details [12]. The final expression for the integral over z entering eq. (11) has the form:

$$\int_0^1 \frac{dz}{z}\,\Phi^{I=0}(z,\zeta,m_{\pi\pi}\approx 2m_\pi) = \int_0^1 \frac{dx}{x} Q(x)\left(\frac{1-x^2}{\sqrt{(1+x)^2-4\zeta x}} + \frac{1-x^2}{\sqrt{(1-x)^2+4\zeta x}} - 2\right), \tag{12}$$

where the function $Q(x)$ is related to the singlet quark distribution in the pion:

$$Q(x) = 2q_\pi(x) - x\int_x^1 \frac{dy}{y^2} q_\pi(y)\,,$$

with

$$q_\pi(x) = \frac{1}{N_f}\sum_f (q_f(x)+\bar q_f(x))$$

Note that the normalization points of $q_\pi(x)$ and $\Phi^{I=0}(z,\zeta,m_{\pi\pi})$ are the same.

Using the eq. (12) and dispersion relations derived in ref. [6] we can write the following expression for the amplitude (11) keeping only $S-$ and $D-$ waves:

$$M_{\mu\nu} = \frac{5}{9}e^2 g^{\perp}_{\mu\nu}\Big[A_0\ f_0(m_{\pi\pi})e^{i\delta_0^0(m_{\pi\pi})}\ P_0(\cos\theta_{\rm cm}) + A_2\ f_2(m_{\pi\pi})e^{i\delta_2^0(m_{\pi\pi})}\ P_2(\cos\theta_{\rm cm})\Big]\,, \tag{13}$$

where the A_0 and A_2 are given in terms of singlet quark distribution in the pion. For A_0 we give the full expression:

$$A_0 = -\frac{1}{\beta}\int_0^1 dx Q(x)\left(\frac{2\beta}{x} + \frac{1-x^2}{x^2}(\sqrt{1-2\beta x+x^2}-\sqrt{1+2\beta x+x^2})\right), \tag{14}$$

where β is the velocity of produced pion in centre of mass frame

$$\beta = \sqrt{1-\frac{4m_\pi^2}{m_{\pi\pi}^2}}\,.$$

The scattering angle in c.m. frame is:

$$\cos\theta_{\rm cm} = \frac{2\zeta-1}{\beta}\,.$$

In two limiting cases: nonrelativistic pions ($\beta\to 0$) and ultrarelativistic ($\beta\to 1$) that we have for A_0:

$$A_0 \;=\; \int_0^1 dx Q(x)\begin{cases} \frac{2}{x}(\frac{1-x^2}{\sqrt{1+x^2}}-1) & \beta\to 0\\ -2x & \beta\to 1\end{cases}$$

The full expression for A_2 is more complicated and we give here only its limiting cases:

$$A_2 \;=\; \int_0^1 dx Q(x)\begin{cases} \frac{2x(1-x^2)}{(1+x^2)^{5/2}}\beta^2 & \beta\to 0\\ 2x(1-x^2) & \beta\to 1\end{cases}$$

The functions $f_{0,2}(m_{\pi\pi})$ in eq. (13) can be related to $\pi\pi$ phase shifts $\delta_0^0(m_{\pi\pi})$ and $\delta_2^0(m_{\pi\pi})$ using Watson theorem and dispersion relations derived in [6]:

$$\log f_l(m_{\pi\pi}) = m_{\pi\pi}^2\left(b_l + \frac{m_{\pi\pi}^2}{\pi}{\rm Re}\int_{4m_\pi^2}^\infty ds\frac{\delta_l^0(s)}{s^2(s-m_{\pi\pi}^2-i0)}\right), \tag{15}$$

where the constants b_l can be estimated using low-energy models of QCD. For example, the estimate in the instanton model of the QCD vacuum gives [6]

$$b_0 = b_2 = \frac{N_c}{48\pi^2 f_\pi^2} \approx 0.73\ {\rm GeV}^{-2}\,.$$

Let us note that the expression (16), strictly speaking, is valid only in the elastic region ($4m_\pi^2 \le m_{\pi\pi}^2 \le 16m_\pi^2$). It is rather easy to extend its range of applicability including the contributions of higher intermediate states (probably the most important is the contribution of $K\bar K$) in the dispersion relations.

3. Conclusions

We showed on two examples of hard exclusive reactions that the measurements of the shape of dipion mass distributions in these reactions can provide us with important information on partonic structure of the pion and two-pion resonances.

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